Optimized GPU-accelerated Monte Carlo program for real-time dose estimation directly using mesh-type computational phantoms*

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Mesh-type phantoms represent the latest generation of human computational phantoms, offering high resolution and adjustability advantages for individualized radiation dosimetry. Current dosimetry computation methods, which require conversion to tetrahedral mesh models for efficient Monte Carlo simulations, still do not meet the requirements for real-time dose calculations. Advancements in heterogeneous computing now allow for significant acceleration in mesh-type phantom calculations by utilizing both high-performance hardware and efficient algorithms. This study aims to develop a GPU-accelerated Monte Carlo simulation method that directly utilizes mesh-type phantoms to further enhance the speed of human dose calculations without the need for tetrahedralization. For the boundary representation polygonal models, this study redesigned and implemented the entire procedural flow of the GPU-accelerated Monte Carlo program, developing particle transport methods within the mesh-type model. All triangular facets of the mesh-type model were constructed into a tree-like acceleration structure and the traversal access pattern was optimized. Moreover, this study adopted an event-based transport method, transporting particles step-by-step by particle type, and a bias-based variance reduction technique employing geometric weights was integrated. For typical external irradiation scenarios, dose calculations between Geant4 and our GPU-based program were compared to assess computational accuracy and efficiency. Compared to the benchmark simulations conducted on a single-thread CPU via Geant4, the organ dose discrepancies calculated by the GPU-accelerated program generally remained within a 5% margin, while computational times were reduced by factors ranging from 500 to 50000. To our knowledge, this study is the first to utilize a mesh-type model for GPU-accelerated dose calculation without tetrahedralization. The simulation time has been dramatically reduced from hours to just mere seconds, offering a rapid and precise Monte Carlo method for mesh-type computational phantoms. This development supports real-time dose calculation studies using dynamic mesh-type models, providing a robust Monte Carlo simulation tool.

Keywords: GPU Monte Carlo, Mesh-type phantom, Heterogeneous, Real-time dose

I. INTRODUCTION

Monte Carlo (MC) simulations employing computational phantoms serve as a crucial method for human dose assessment. The boundary-represented mesh-type models, as the latest generation of computational phantoms, offer the dual benefits of flexible deformability and high resolution, which better represent the dosimetric characteristics of real human body [1–3]. Consequently, mesh-type phantoms have demonstrated significant potential in the fields of radiation therapy, radiation protection, and individual dosimetry, where more accurate models are essential for obtaining precise individual doses [4–7].

However, employing mesh models directly in MC simulations for dose calculations introduces challenges [8]. In MC simulations, accurately defining the relationships between particles and their surrounding geometrical structures is esrounding sential [9, 10]. The mesh-type phantoms used for MC simulations are typically composed of polygonal grids, primarily based on triangular meshes [2]. Other polygonal mesh models can also be easily subdivided into triangular meshes. Direct use of the mesh-type phantom containing a large number

22 of triangular facets for these computations notably reduces 23 computational speed, with research indicating that such use increases computational time by 70 to 150 times compared 25 to voxel models [11]. In response, a novel method involving tetrahedral decomposition [12, 13] has been developed 27 to expedite computations for mesh phantoms [14, 15], and 28 the resulting tetrahedral models also support posture adjust-29 ments [16]. While this tetrahedralization simplifies the determination of geometric relationships during particle transport, the subdivision of the mesh into tetrahedra substantially increases the internal complexity, which impedes further acceleration of computation speeds [12]. Thus, in fields like clinical radiotherapy, nuclear medicine, and accident dose reconstruction, where strict time constraints are crucial, the computational time of mesh-type models often exceeds ac-37 ceptable limits, restricting their applicability in these critical 38 areas [17, 18].

In recent years, the rapid development of GPU hard-ware and continual optimization of ray-tracing software algorithms have made it feasible to directly employ boundary-represented models for dose calculations [19–22]. GPUs, with their superior floating-point computational capabilities and increased thread count, are better suited for large-scale particle simulations compared to CPUs [23, 24]. Consequently, numerous GPU-accelerated programs for photon and photon-electron coupled transport have been developed, achieving significant acceleration [18, 25–29]. Our group has

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49 previously developed the first GPU-accelerated MC transport 104 tion methods in accelerating the simulations was assessed. 50 program based on tetrahedral phantoms. However, due to is-51 sues with GPU thread divergence [30] and the increased internal complexity caused by tetrahedralization [12], the com- 105 A. GPU-based Monte Carlo program for mesh-type phantom putational time of the tetrahedral GPU-accelerated MC program remains at the level of tens of seconds to minutes. Once 106 the direct computation with mesh-type phantoms is achieved, 56 bypassing the tetrahedral segmentation step and addressing 57 the slow particle-to-geometry positioning, there is potential 58 for further improvements in computational efficiency. To the 59 best of our knowledge, current popular GPU MC programs 60 have not yet implemented GPU acceleration directly based on 61 mesh-type models. The principal challenge lies in the com-62 plexity of implementing rapid particle transport within mesh-63 type models on GPUs [31]. Additionally, GPU-based MC 64 programs often face significant thread divergence issues due 65 to the considerable variability among different particle trans-66 port processes [30, 32]. Furthermore, smaller organs tend to 67 present larger statistical errors in dose results because of the 68 lower probability of particle interactions [33], necessitating 69 an increase in the number of simulation particles to achieve 70 more accurate dosimetry, which significantly extends the sim-71 ulation time.

To further enhance the computation speed for dose calcu-73 lation in mesh-type phantoms, this study implemented GPU-74 based MC simulation program directly utilizing boundary-75 represented mesh models. This involved redesigning the en-76 tire procedural flow of particle transport. All triangular facets the mesh model were organized into a tree-like accel-78 eration structure and the traversal access pattern was opti-79 mized, significantly reducing the complexity of geometric lo-80 calization and intersection calculations during particle trans-81 port. Furthermore, this study adopted an event-based trans-82 port method, conducting multiple simulations in which parti-83 cles of the same type were transported for a single step dur-84 ing each simulation, rather than relying on a single thread 85 to transport one particle until termination, which greatly re-86 duced thread divergence and improved hardware utilization. 87 The introduction of multi-GPU parallel processing further accelerated the computation speed. Additionally, we employed 89 a biasing sampling technique based on geometric weights for 90 variance reduction, significantly reducing statistical errors in smaller organs and decreasing the number of simulated par-92 ticles. Rigorous validation of the program demonstrated pre-93 cise computational outcomes and substantial acceleration, ef-94 fectively addressing the challenges associated with enhancing 95 dose calculation speed for mesh-type phantoms.

MATERIAL AND METHODS

MC simulations. Various optimization techniques, including 153 enhancing overall intersection efficiency. event-based transport, multi-GPU parallelism, and variance 154 102 the computational speed. Finally, the accuracy of the GPU 156 structure. This complexity presents significant challenges for ₁₀₃ program was validated, and the effectiveness of the optimiza-₁₅₇ GPU computing, which is limited by stack depth [31]. Such

1. Constructing flat acceleration structure

In MC simulations, determining both the physical and ge-108 ometric step sizes requires the material cross-section information at the particle's location, as well as the distance to the boundary along the particle's trajectory [9, 10, 34]. For meshtype phantoms, it is necessary to traverse all triangular facets, 112 to perform particle localization and intersection operations. The repetitive traversal operation, which must be performed 114 for each particle transport step, is time-consuming. To en-115 hance traversal efficiency, the implementation of acceleration 116 structures becomes crucial [35–37]. These structures system-117 atically organize data into layers, significantly reducing the 118 number of searches and enhancing query efficiency. Given this context, we construct a tree-based acceleration structure 120 for all triangular facets of the mesh-type phantom, to substan-121 tially reduce the time complexity of data traversal. Consider-122 ing both time complexity and the need for dynamic updates beneficial for phantom adjustments [38], we choose to im-124 plement a Bounding Volume Hierarchy (BVH) tree, which 125 is widely used in ray tracing and animation [39]. While 126 the BVH acceleration structure can theoretically handle other 127 polygonal elements, such as polygons with arbitrary shapes as leaf nodes, we primarily focus on triangular meshes due to 129 their prevalence in human phantoms, like the ICRP's MRCP 130 model. Furthermore, using triangular facets simplifies classi-131 fication and traversal, making it more efficient and suited to 132 the requirements of dose calculations.

The constructing process of BVH tree begins with calculating a bounding box for all triangles, which forms the parent 135 node [39]. These triangles are then divided into two groups 136 based on a specific pattern, such as an average division by 137 quantity. The bounding box calculation and division process 138 continue for the two child nodes until the number of triangles in a subdivided child node falls below a specified threshold, at which point it becomes a leaf node and stores the information 141 of the included triangles, as illustrated in figure 1. An ef-142 fective partitioning strategy is essential. When geometric ob-143 jects in a scene are unevenly distributed, traditional partition-144 ing methods may yield an unbalanced tree structure, which can reduce traversal efficiency. To optimize this process, the Surface Area Heuristic (SAH) is typically employed [39, 40]. The SAH evaluates the cost of each partition by calculating the surface area of the resulting child bounding boxes. This 149 method explores various partitioning schemes and selects the one with the lowest cost for implementation. Such optimiza-This study presents the development of a program that 151 tion can significantly reduce unnecessary intersection tests directly utilizes a mesh-type model for GPU-accelerated 152 between rays and bounding boxes during ray tracing, thereby

In mesh-type phantoms, the presence of numerous triangureduction methods, were then implemented to further enhance 155 lar facets results in a deeply nested and complex acceleration

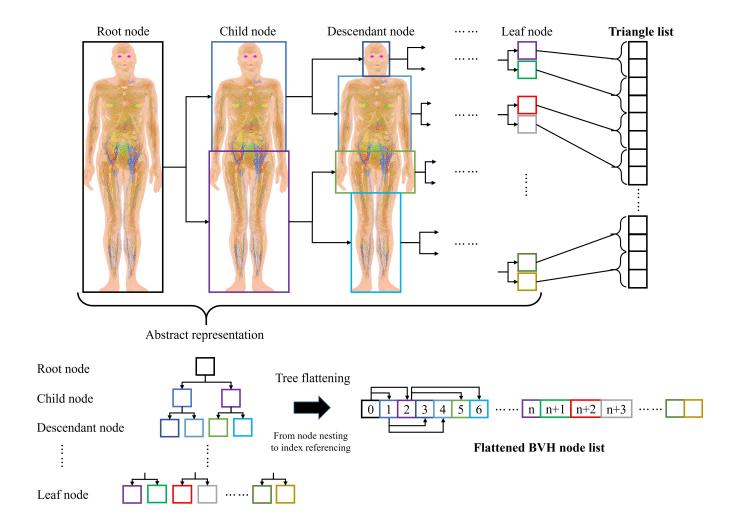


Fig. 1. (Color online) Flowchart for constructing flat acceleration structure of the ICRP mesh-type phantom. The upper part of the diagram shows the step-by-step division of the surface model into bounding boxes to build a tree structure. This division continues until the number of triangles within a bounding box falls below a threshold, at which point the division stops, and the node becomes a leaf node, storing the indices of the encompassed triangles. The lower part of the diagram provides an abstract description, followed by tree flattening, where child nodes are referenced in the node list by their indices instead of node nesting.

158 constraints restrict the number of recursive calls and the depth 176 ficiently processed on GPUs. 159 of data structures that can be processed, adversely affect-160 ing the performance of recursive algorithms traversing these structures. To overcome this limitation, the flattening of acceleration structure becomes essential. This process involves 177 2. Transport in the acceleration structure composed of numerous 163 converting the tree structure into a linear format for efficient 164 processing on GPUs [41]. Our implemented "tree flattening" 165 method is illustrated in figure 1. We record the indices of 179 166 child nodes separately in the node sequence rather than creat- 180 depicted in figure 2. Particles are generated through sam-167 ing new child nodes directly within each node. This strategy 181 pling on the GPU and subsequently transported in a stepwise 168 produces two sequences: one containing a list of BVH nodes 182 manner until termination [9]. As previously noted, localizathat sequentially stores information for each node, and an- 183 tion and intersection operations represent the most critical and 170 other comprising a list of triangular facets that sequentially 184 time-consuming aspects of particle transport. Localization is 171 records the geometric and material information associated 185 necessary to obtain the material information of the geometry 172 with the leaf nodes. The flattening of the BVH tree struc- 186 where the particle resides, allowing for energy interpolation 173 ture circumvents the limited stack depth of GPUs by avoiding 187 to derive the reaction cross-sections and sample the physi-174 nested configurations. This architectural adjustment ensures 188 cal step length. The intersection operation determines the

triangle

The particle transport process within the mesh model is 175 that more complex phantom acceleration structures can be ef- 189 distance to the nearest geometric boundary in the direction 190 of particle motion, thereby determining the geometric step

191 length. By comparing the physical step lengths associated 192 with various reactions to the geometric step length for trans-193 port, the minimum value is selected to ascertain whether a 194 reaction occurs or the particle is transported to the bound-195 ary [9, 42]. This process is repeated until the particle and its 196 secondary particles are terminated.

Determining the relationship between particles position 198 and mesh structures typically involves emitting a virtual ray 199 from the particle, often aligned with the coordinate axes, as 200 illustrated in figure 3(a). The number of intersections and 201 the corresponding distances are used to ascertain the current 202 geometry and material of the particle. Generally, the geome-203 try with an odd number of intersections that is closest to the 204 particle is selected. Calculating the geometric step length re-205 quires determining the distance to the nearest grid boundary along the particle's direction of motion, which also necessitates intersection operations. By integrating these two processes, both particle localization and intersection detection can be accomplished simultaneously, as shown in figure 3(b). 210 To further enhance the efficiency of these procedures, we pre-211 assign additional information to the triangular facets of the 212 mesh model. Specifically, we add the normal vector for each 213 triangular facet, indicating the outward direction from the in-214 terior of the mesh geometry, and specify the materials asso-215 ciated with the outward and inward normal. By identify the 216 nearest intersection point and intersecting triangle, we can de-217 termine the material in which the particle is currently located 218 by assessing the relationship between the particle's motion 219 direction and the triangle's normal direction, as depicted in 220 figure 3(c). This approach eliminates the need to count intersections and assess the number of intersection points with the same geometry, thereby reducing computational time.

The preceding discussion focused on the transport of par-224 ticles after their intersection; however, it did not detail the 225 methods for quickly intersecting a large number of triangles based on particle position and directional information. This is where the acceleration structure we previously constructed becomes essential. We will establish an empty stack of intersection nodes, placing the first node of the flat acceleration structure node sequence at the top of this stack. Next, we will fetch the top node from the stack. If this node is not a 247 leaf node, we will perform an intersection operation between the particle's direction of motion and the two child bounding 248 boxes of this node. If an intersection occurs, the correspond- 249 volve the physical processes of coupled photon-electron 295 ing child nodes of the intersecting bounding boxes will be 250 transport, the GPU MC program developed in this study pro-236 added to the top of the stack. If the fetched node is a leaf node, 251 vides a comprehensive simulation of these coupled transwe will traverse the few triangular facets within it to per- 252 port processes, utilizing the same physical models and cross-238 form ray-triangle intersections, recording information about 253 sectional data previously researched by our team [43]. This 239 any intersecting triangles. This process continues until the 254 GPU program effectively simulates the physical interactions stack is empty, at which point we select the shortest intersec- 255 for photons and electrons across an energy range from 0.001 tion distance as the geometric step length and determine the 256 MeV to 100 MeV. For photons, photoelectric absorption, 242 intersecting triangle. Subsequently, we will use the material 257 Compton scattering, and pair production are considered. 243 information associated with the inward and outward normal 258 For electrons and positrons, the physical processes include 244 to identify the material at the particle's position. Pseudocode 259 bremsstrahlung radiation, ionization effects, multiple scat-

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Algorithm 1: Particle intersection and localization
within acceleration structure
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```
Input: Particle position, particle direction, and
             acceleration structure
     Output: Result of intersection and localization
   1 Initialize an empty stack of nodes;
   2 Push the root node onto the stack;
   3 min\_distance = \infty;
     while stack is not empty do
         node = Pop top node from stack;
   5
         if node is a leaf node then
   6
   7
             for each triangle in node.triangles do
                 Check if particle intersects with the
   8
                  triangle;
                 if intersection distance < min_distance
    9
                  then
                     min\_distance = intersection distance;
   10
                     Update intersecting triangle;
                 end
   12
             end
   13
246
   14
         else
             if particle intersects with the bounding box of
              left child node then
                 Push left child node onto the stack;
   16
   17
             if particle intersects with the bounding box of
   18
              right child node then
                 Push right child node onto the stack;
   19
             end
   20
         end
  21
  22 end
  23 if intersection exists then
  24
         triangle = Get intersecting triangle;
         material =
  25
          dot product(triangle.normal, particle.direction) >
          0? triangle.inMat: triangle.outMat;
         geometric\_step\_length = min\_distance;
  26
  27 else
         Kill particle;
  28
```

Program implementation framework

29 end

Given that dosimetry simulations in the human body in-245 illustrating this more intuitively is shown in the algorithm 1. 260 tering, and positron annihilation. Furthermore, the founda-

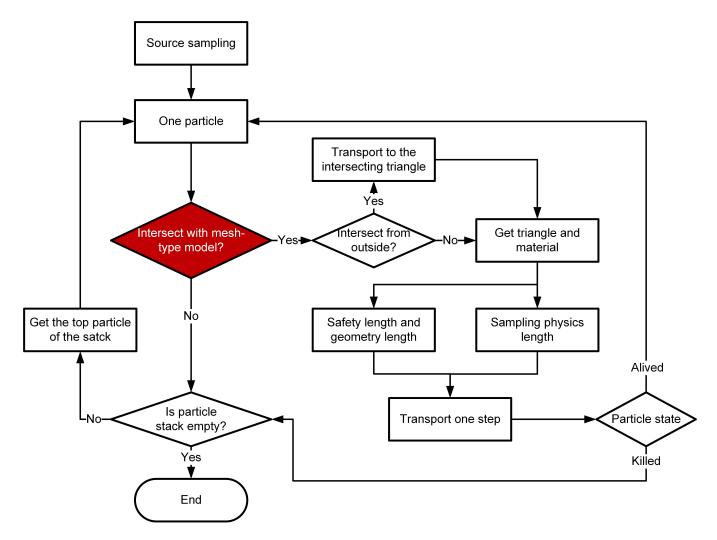


Fig. 2. (Color online) Particle transport process within the mesh-type phantom composed of numerous triangle.

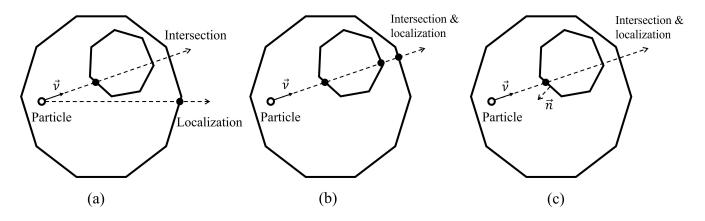


Fig. 3. (Color online) The optimization of particle positioning and intersection operations within the mesh-type phantom. Subfigure (a) illustrates the separate execution of particle positioning and intersection operations. Subfigure (b) demonstrates that the determination of particle positioning and intersection is achieved through the particle direction and geometric intersections. Subfigure (c) shows the evaluation of particle position based on the normal information of the nearest intersecting triangle and the particle direction.

tional framework and initialization procedures remain consis- 263 els and construction processes for acceleration structure. On 262 tent, requiring only the integration of the reading mesh mod- 264 the GPU side, particle transport employs the transport method

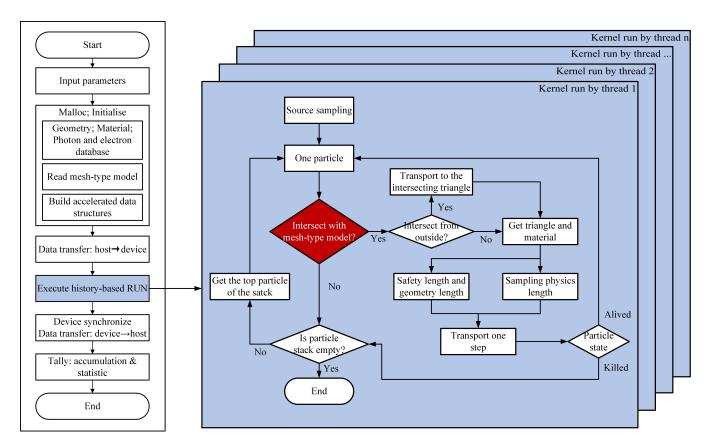


Fig. 4. (Color online) The history-based transport workflow of the GPU MC program using the mesh model. Modules within the blue areas indicate operations performed on the GPU.

265 described in the previous section. The workflow of the GPU 287 performance. This phenomenon is particularly prevalent in 266 MC program for photon and electron transport using the mesh 288 GPU MC simulations. Traditional MC particle transport almodel is illustrated in figure 4. A history-based transport 289 gorithms typically rely on historical transport methods, akin 268 method is utilized, in which each GPU thread simulates a sin- 290 to the GPU MC program implemented in the previous secfar, we have successfully developed a GPU MC program for 292 particle until its lifecycle concludes, as illustrated in figure dose calculation that directly utilizes the mesh model. The 293 4. However, during the simulation, the transport path lengths subsequent sections will focus on optimizing and enhancing 294 and times of different particles can vary significantly [32, 45]. 273 this program.

Event-based transport method

Solving thread divergence issues

275

276

groups of 32, known as "warps", for efficient management 303 ticle processes into distinct event types. Each GPU batch and scheduling. Within a warp, all 32 threads execute in- 304 processes only one type of event at a time, and then cycles structions in a lock-step manner, meaning that any instruction 305 through them. This approach ensures that all GPU threads to be executed by any thread in the warp must be performed 306 execute the same type of event simultaneously, such as percountered, the entire warp executes the whole branches taken 308 the transport is completed, the initial or generated secondary by any thread. In such cases, if different threads within the 309 particles are reallocated to sort and consolidate the surviving 284 warp follow distinct code branches, thread divergence oc- 310 particles, followed by a subsequent execution. As illustrated 285 curs. Although divergence does not affect the correctness of 311 in figure 5, this method significantly reduces the divergence 286 individual thread computations, it significantly impacts code 312 of GPU threads, thereby improving hardware utilization and

gle particle from its generation to termination [9, 32]. Thus 291 tion, where each thread simulates the transport of a single 295 For instance, some particles may be "killed" after one single step, while others may require multiple transport steps, as shown in figure 5. Consequently, substantial code branching often occurs when different threads within the same warp handle particles with vastly different transport characteristics, leading to severe thread divergence.

To address this issue, relevant studies have proposed an GPUs contain numerous threads, which are organized into 302 event-based transport method [32, 46, 47], which divides parsimultaneously [32, 44]. When a branching instruction is en- 307 forming a single step of photon geometrical transport. Once 313 computational efficiency.

Implementation method in the GPU program

Building upon the GPU-based MC program developed in 370 $_{316}$ the previous section using the mesh-type model, we extend its 371 317 functionality to implement an event-based transport method. 372 The implementation process is illustrated in the figure 6. We 373 tion and data transfer processes. The updated code framework modify the previous approach of looping through the transport steps of a single particle by removing the loop for particle transport within the GPU kernel function and replacing it 376 is invoked automatically when a new class object is created with a stepwise transport kernel method. To facilitate classification by particle type, we prepare an event particle library in advance, which stores the particles after each batch transport step. Furthermore, we establish multiple sequences for the 380 date module can be executed independently before each simdifferent events to record particle indices in the event particle 381 ulation to modify source information and update geometric 327 library. The division of particle transport processes for pho- 382 data. This module involves a smaller data volume and also 328 tons, electrons, and positrons allows us to invoke the GPU 329 MC program to transport the particle type with the largest 384 component, the GPU particle transport module, utilizes multicount at each step, as the reaction sampling for the same par- 385 threading to start particle simulation on corresponding GPUs. ticle type is generally consistent, resulting in minimal thread 386 Considering the significant computational power discrepan-332 divergence. Upon completion of one transport step, we cat-233 egorize the current particles (if still alive) and the generated 288 assigns simulation particle counts based on the number of 334 secondary particles according to their types and repeatedly invoke the GPU transport program until the number of particles 390 GPU has been incorporated in this module, as shown in figure $_{336}$ in the event particle library to be simulated falls below a pre- $_{391}$ 7(b). 337 determined threshold. The event-based transport method introduces additional operations compared to the history-based 339 approach, such as multiple GPU kernel function calls and the 392 340 need for particle sorting and data synchronization, which in- 393 341 curs extra time costs. Consequently, when the particle count 342 is low, continuing with the aforementioned event-based transport may reduce efficiency. Therefore, we opt to transport the 395 reliable when statistical errors are relatively small [2]. How-344 remaining particles below a predetermined threshold directly 345 using the history-based method until they are terminated.

C. Multi-GPU parallel optimization

348 strained by the number of threads, we considered utilizing 403 that increase particle transport in smaller organs, the total its through an increase in hardware. In the previous single 405 cantly reduced, enhancing computational efficiency. GPU workflow, the CPU reads and initializes data, which 406 are returned to the host for post-processing. Simply replicat- 409 quantities. Among these, the region importance biasing 355 ing this process via multi-threading on the CPU, where each 410 method is particularly prevalent [48]. The central principle of thread independently executes the full workflow using differ- 411 this method is to assign higher importance to regions of intertentially exceeding computer memory limits if many GPUs 416 optimal variance reduction can be achieved by precisely set-362 are employed. Moreover, each new simulation iteration, such 417 ting the bias parameters. Given the fixed structure of human 363 as updating source term information, requires the complete 418 models and the consistent size of internal organs, employing 364 re-execution of this workflow, leading to considerable delays. 419 regional importance sampling is appropriate for addressing

To achieve more efficient multi-GPU parallelism, we trans-366 formed the executable simulation program into a library 367 named "Simulation", followed by a restructuring of the code 368 architecture into independently executable modules. By loading this library in the main program and creating an instance of the simulation class, different functions can be invoked through this object to operate the modules separately. This approach significantly reduces the redundancy of initializa-375 ded in the constructor of the simulation class. This module and is responsible for initializing the physical and geometri-378 cal models. This process is executed only once and the data is 379 subsequently transferred to the corresponding GPU. The up-383 requires transferring the updated data to each GPU. The final 387 cies among different GPUs, a load balancing mechanism that 389 CUDA(Compute Unified Device Architecture) cores in each

Implementation of variance reduction in the GPU MC program

In human organ dose simulations, results are considered 396 ever, due to significant variations in the size, position, and 397 shape of different organs, statistical errors also vary consid-398 erably. Typically, smaller organs are less likely to be reached 399 by particles, making these interactions rare and requiring a 400 large number of simulated particles for accurate results [33]. 401 In contrast, larger organs do not require such extensive parti-Due to the limitations of single GPU simulations con- 402 cle simulations. By employing variance reduction techniques multiple GPUs to overcome the current acceleration lim- 404 number of particles required for simulations can be signifi-

Biasing variance reduction techniques are widely used in is then transferred to the device, followed by the GPU ex- 407 mainstream MC simulation programs to decrease computaecuting the simulation computations, and finally, the results 408 tional variance by artificially adjusting particle weights and ent GPUs, would result in redundant initializations. This re- 412 est and lower importance to less critical areas. Modifications dundancy is inefficient due to the substantial size of the phan- 413 of particle numbers and weights can be achieved through tom data and physical model cross-sections, which can sig- 414 strategies such as surface splitting and Russian roulette based nificantly increase processing time and memory usage, po- 415 on the importance assigned to these regions. Theoretically,

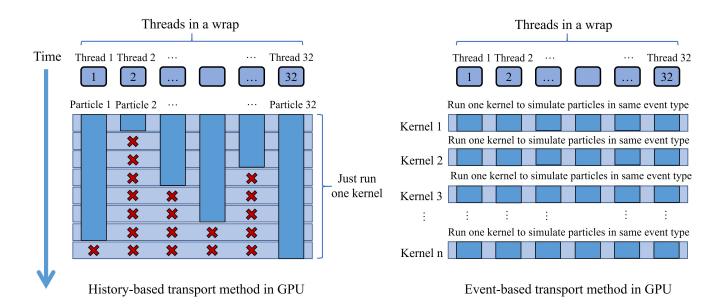


Fig. 5. (Color online) Comparison of history-based and event-based transport methods on the GPU. The deep blue bars represent the duration of particle traversal. By categorizing event types and executing the same events through multiple kernel calls, the variability in execution across different threads is significantly reduced, thereby minimizing thread divergence.

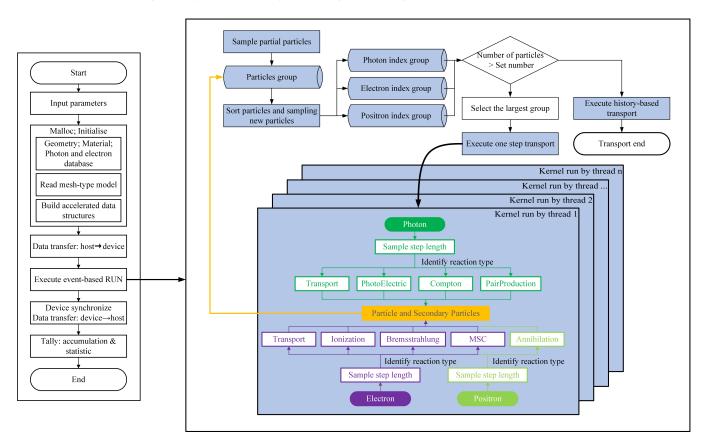


Fig. 6. (Color online) The implementation process of event-based transport method in the GPU MC program for mesh-type phantom. Modules within the blue areas indicate operations performed on the GPU.

the disproportionate consumption of particles in obtaining acturate smaller organ dose.

Based on the foundation of this work's GPU MC simulation program, the process to implement variance reduction

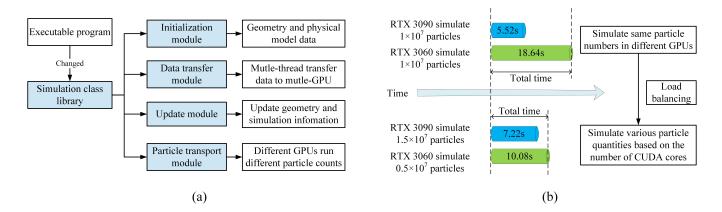


Fig. 7. (Color online) Schematic of Multi-GPU parallel optimization. Subfigure (a) shows the basic modules of the simulation class. Subfigure (b) illustrates effect of load balancing; the upper part displays the performance when different GPUs run the same number of particles, while the lower part shows performance when the number of particles is adjusted according to each GPU's capabilities.

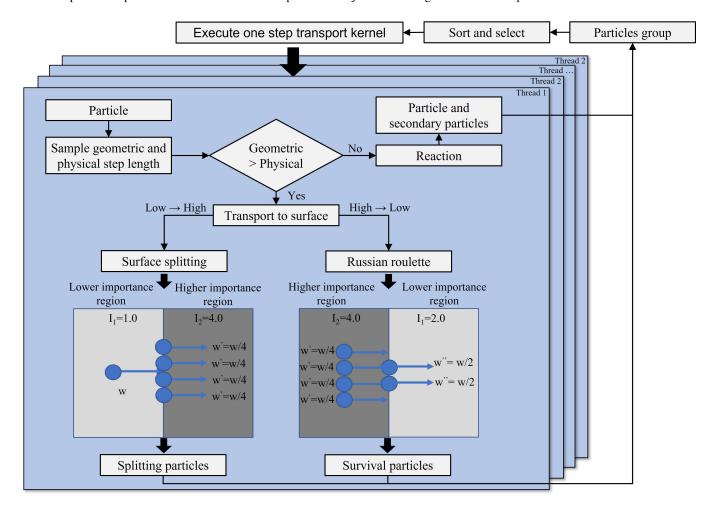


Fig. 8. (Color online) The implementation of variance reduction techniques using regional importance method in the event-based GPU MC program. The areas shaded in blue indicate operations executed on the GPU, while the operations outside the blue areas are performed on the CPU.

techniques through regional importance method is depicted traverse geometric boundaries, they undergo transfortransfor-des in figure 8. This process involves assigning importance parameters to both the geometries and the particles. As par-des terms to both the geometries and the particles are depending on the relative im430 portance of the current and subsequent geometries. Surface 485 those of the benchmark. 431 splitting generates multiple identical particles at the same lo-486 432 cation, which are then included in the particle sequence for 487 mesh-type models compared to tetrahedral meshes, the CPU 433 simulation. Roulette involves a probabilistic termination of 488 benchmark in this study employs a tetrahedral model for com-434 particles, wherein a particle is eliminated if a randomly drawn 489 parison. In contrast, the GPU simulations directly utilize number exceeds the ratio of the importance values between 490 mesh-type phantoms. The simulation outcomes are the exterthe upcoming and current geometries. In the tally module, 491 nal radiation absorbed dose conversion coefficients for varienergy deposition must be weighted based on the current par- 492 ous organs. 437 438 ticle's importance.

439 ance reduction techniques, offers an intuitive principle and 495 108 particles with history-based transport method, (3) GPU rational design of importance or weight. To this end, sim- 497 (4) GPU simulation 10⁷ particles with event-based transport ulations based on actual irradiation scenarios can be con- 498 and variance reduction, and (5) GPU simulation 10⁷ particles ducted in advance, with iterative adjustments to region im- 499 with event-based transport method, variance reduction and portance for achieving lower statistical errors of organ doses 500 multi-GPU mode. The objective was to evaluate the dose calwith fewer particles [49].

E. Dose calculations and efficiency assessments

To assess the accuracy and acceleration efficiency of our 449 GPU-based program, we conducted simulations on a standard 504 450 external irradiation scenario (anteroposterior, AP) using the 451 ICRP's Mesh-type Reference Computational Phantoms (MR- 505 452 CPs) [2]. The male phantom of MRCPs was subjected to 506 and four GPU simulation methodologies incorporating vari-453 unidirectional and parallel photon and electron beams emit- 507 ous optimization techniques. The bar graph in figure 9 repre-454 ted from a planar source, with a monoenergetic energy of 10 508 sents the absorbed doses in various organs per fluence of pho-455 MeV. The choice of this energy was driven by the higher po- 509 tons with 10 MeV energy under AP irradiation using different 456 tential risks associated with high-energy particles, as well as 510 simulation methods. Given that all four GPU simulation con-457 the longer computation time and more complex interactions 511 figurations are distinct modalities of our GPU MC program, involved. The organ materials in the simulations were defined 512 and can be selected based on specific hardware conditions 459 from the ICRP Publication 145. 460

461 462 CPU platform, adapting the source code from the supplemen- 516 four GPU methods compared to the CPU benchmark for each 470 specifications included 64 GB RAM(Random Access Mem- 524 benchmark, thus confirming the accuracy of the GPU proory) and an Intel(R) Xeon(R) CPU E5-2660 v4 @ 2.00GHz 525 gram's calculations. which has 28 threads.

474 type phantom, instead of the tetrahedron model, was di- 528 tion techniques, figure 10 employs a box-and-whisker plot to 475 rectly employed for the same irradiation scenario simula- 529 illustrate the distribution of dose deviations for all organs in 476 tion. To further assess GPU acceleration, we conducted 530 a mesh-type phantom across different GPU calculation methtests to compare computation results and times both with 531 ods compared to the benchmark. Each box-and-whisker (coland without event-based transport, variance reduction tech- 532 umn) in figure 10 represents the distribution of dose simuniques, and multi-GPU configurations. These simulations 533 lation results for a particular GPU simulation scenario, with were performed on an NVIDIA GeForce RTX 4090 GPU, 534 each point indicating the relative deviation of each organ's which boasts 24 GB VRAM(Video Random Access Mem- 505 results from the CPU benchmark. The box region represents 482 ory), operates on CUDA version 12.2 and features 16,384 536 the range of relative deviations for the half of organs. The 483 CUDA cores. The secondary electron and photon energy 537 narrower the box-and-whisker and closer its range is to zero, 484 thresholds and the irradiation scenario were consistent with 538 the more accurate the overall simulation. Notably, to bet-

Due to the slower performance of CPU calculations with

In summary, we simulated five configurations: (1) CPU Region importance sampling, compared to other vari- 494 simulation 108 particles (benchmark), (2) GPU simulation straightforward implementation. However, the key lies in the 496 simulation 108 particles with event-based transport method, 501 culation accuracy and the acceleration efficiency of the GPU 502 program.

RESULTS AND DISCUSSION

Comparison of calculated dose values

Figure 9 illustrates the dose results from a CPU benchmark using mass fractions of various elements, with data sourced 513 and simulation requirements, it is imperative to ensure the 514 accuracy of these four computational results. We exhibit the For benchmarking, we employed Geant4 simulations on a 515 maximum relative deviations in organ doses calculated by the tal material of the 145th publication for this AP irradiation 517 organ. The accompanying line graph displays the maximum scenario [2]. Notably, considering the slow dose calculation 518 deviation. Statistical uncertainty calculations in our GPU prospeed with the mesh model on CPU, we employed the tetrahe- 519 gram are conducted using batch-based method [17, 50, 51]. dral model for simulation [8, 11]. Our computational setup in- 520 The statistical error for most organs across the five simulacluded Geant4 version 10.04, utilizing the Livermore physics 521 tion methods is within 3%. As shown in figure 9, the results model with secondary electron and photon energy thresholds 522 for the majority of sensitive organs are consistent across all of 0.2 MeV and 0.002 MeV, respectively. The CPU hardware 523 five calculation methods, with discrepancies within 5% of the

To further analyse the variations among different GPU sim-For the GPU program developed by this study, the mesh- 527 ulation methods, particularly the effects of various optimiza-

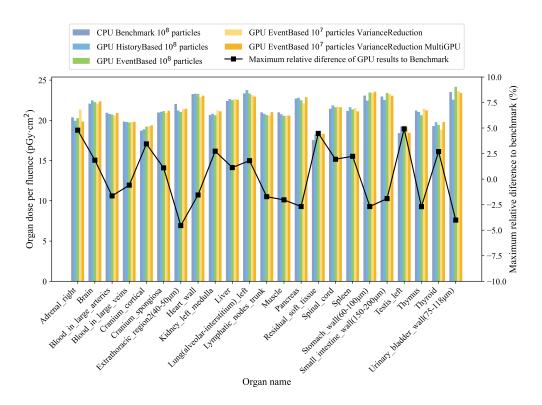


Fig. 9. (Color online) Comparison of organ equivalent doses per fluence of 10 MeV photon under AP irradiation across different simulation configurations. The bar charts represent the dose calculation results for each simulation configuration, while the line graph shows the maximum deviation of GPU-calculated results for each organ compared to the CPU benchmark.

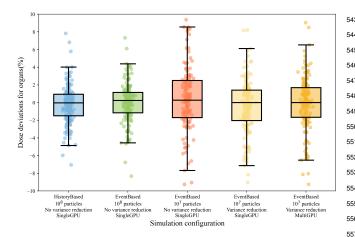


Fig. 10. (Color online) Distribution of dose deviations for all organs of the mesh-type phantom across different GPU simulation configurations compared to the benchmark results. The box represents the interquartile range (from the first quartile to the third quartile). The excluding some outliers.

539 ter demonstrate the impact of variance reduction optimiza- 565 tion methods, the analysis includes simulations using only the event-based method with 10^7 particles.

ulations utilizing with 10^8 particles, show that the relative deviations in dose calculations for most organs are within 3%. This consistency arises because the history-based and eventbased methods do not fundamentally differ in their physical models, but rather in their transport mechanisms. A comparison of these two 10^8 particle simulations (left two box plots) with the three 10^7 particle simulations (right three box plots) demonstrates that simulations with 10⁸ particles exhibit noticeably smaller relative deviations. This observation aligns with statistical principles, where a higher particle count leads to lower statistical errors and more precise calculations.

Subsequent comparisons of three GPU simulation meth- $_{555}$ ods, each running with 10^7 particles (the right three box 556 plots), indicate that the range of relative dose differences for ₅₅₇ all organs narrows with the application of variance reduction 558 techniques. The results show that applying variance reduc-559 tion techniques improved the accuracy of simulations with the same particle count, demonstrating the effectiveness of 561 the method, although the accuracy still does not reach that median line inside the box indicates the middle value of the data. 562 of simulations using 108 particles. Nevertheless, these dif-The whiskers extend from the box to show the full range of the data, 563 ferences remain within acceptable limits, but significantly re-564 duce the number of simulation particles.

Comparison of computation speed

Table 1 summarizes the differences in computation times The left two columns of the figure 10, which represent sim- 567 across various simulation configurations. The CPU bench-

11 BEE 1. Comparation times deless various simulation comigarations.					
Program and method	Phantom type	Hardware	Total particles	Simulation	Time ratio to
			simulated	time (s)	benchmark
CPU Single-core	Tetrahedron-type	Intel Xeon CPU E5-2660	1×10^{8}	193322.12	1.00
GPU + History-based	Mesh-type	RTX 4090	1×10^{8}	338.15	571.71
GPU + Event-based	Mesh-type	RTX 4090	1×10^{8}	96.28	2007.92
GPU + Event-based	Mesh-type	RTX 4090	1×10^{7}	10.34	18696.53
GPU + Event-based + Variance	Mesh-type	RTX 4090	1×10^{7}	12.58	15367.42
reduction					
GPU + Event-based + Variance	Mesh-type	$5 \times RTX 4090$	1×10^{7}	3.74	51690.41
reduction + MultiGPU					

TABLE 1. Computation times across various simulation configurations.

568 mark time represents the computation time using a single- 613 569 core for simulation. The term "Time ratio to benchmark" 614 a file size and number of triangular facets approximately an 570 indicates the ratio of other simulation computation times to 615 order of magnitude larger than the Chinese Reference Adult the CPU benchmark time.

573 els developed in this study, when operated without event- 618 are crucial for accurately representing the anatomical strucbased transport or variance reduction techniques, reduces 619 ture, they may not always be essential for dose estimation 575 computation time by a substantial factor of 570 compared 620 purposes. In fact, using the same GPU MC program under the 576 to the single-core CPU MC program. This improvement is 621 same conditions (10 MeV photon AP irradiation with 5 RTX ₅₇₇ due to the lack of acceleration structures or efficient inter-₆₂₂ 4090 GPUs, simulating 10⁷ particles combined with variance 578 section algorithms in conventional CPU MC programs such 623 reduction techniques and event-based transport), the simula-579 as Geant4, whether using mesh-type models or parametric 624 tion time for our CRAM phantom has already been reduced to 580 tetrahedral models. Therefore, combining GPU hardware 625 0.78 seconds, which can essentially be considered as achievwith software optimization algorithms to accelerate mesh- 626 ing real-time dose calculations. 582 type models achieves faster acceleration than the CPU with 627 583 tetrahedral models, and even more so compared to the CPU 628 rates various acceleration and optimization methods, can sigmesh-type models are not mainstream, they are not compared 600 dose calculations to the order of second, achieving up to a here. 586

Employing optimization techniques, such as the event- 632 program. 588 based transport method, further enhances performance, achieving an additional fourfold acceleration when simulating the same number of particles, which results in a total 633 speedup of 2000 times compared to the CPU benchmark. This improvement is primarily because the event-based transport method significantly reduces thread divergence and increases hardware utilization. However, it should be noted that the need for repetitive kernel function calls and additional operations like particle sorting may slightly decrease computational efficiency. 597

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Furthermore, utilizing both event-based transport and variance reduction techniques with fewer particles decreases computation time by 18,000 times compared to the baseline, while the relative differences in organ doses remain within acceptable ranges. However, the implementation of variance reduction techniques, which includes operations such as particle splitting and roulette, can lead to an increase in compu-605 tation time.

GPUs in parallel. This approach involves synchronizing data 649 from hours to seconds, achieving up to a 50,000-fold reducacross different devices. However, due to the varying operational conditions of each device, there can be discrepancies 651 more, the GPU-accelerated calculation method for mesh-type 610 in runtime, which means that the overall simulation compu- 652 phantoms, integrated with human posture capture and defortation time does not decrease proportionally with the number 653 mation technologies, enables real-time human dose calcula-612 of GPUs used.

Considering that the MRCP used is highly detailed, with 616 Male (CRAM) phantom developed by our team [52], it's im-It reveals that the GPU-based program for mesh-type mod- 617 portant to note that while these detailed features of MRCP

In summary, the GPU-based MC program, which incorpowith mesh-type models. Since methods involving CPUs with 629 nificantly shorten the computation times for human phantom 631 50,000-fold reduction compared to the single-core CPU MC

CONCLUSIONS

This study presents the first instance of a GPU-accelerated 635 MC program that performs dose calculations directly using 636 mesh-type models without requiring conversion processes 637 such as tetrahedralization or voxelization. By directly pro-638 cessing polygonal models for dose calculations, the program 639 leverages their flexible deformation capabilities and high res-640 olution, yielding more accurate dosimetric outcomes. Ad-641 ditionally, this approach eliminates the need for tetrahedral 642 conversion and mesh repair associated with tetrahedral mesh-643 type phantoms, thereby simplifying the computational work-644 flow. By incorporating GPU acceleration for MC trans-645 port, constructing acceleration structures, enabling single-646 step transport based on particle type, and applying variance 647 reduction techniques along with multi-GPU parallel opti-Further acceleration can be achieved by utilizing multiple 648 mization, the simulation time for a single phantom reduces 650 tion compared to the single-core CPU MC program. Further-654 tions. This capability is particularly valuable in fields requir655 ing rapid responses, such as occupational exposures, radio- 660 For instance, it currently transports particles based on their 656 therapy, accident dose reconstruction, and individual dosime- 661 type. Future implementations that transport based on dif-657 try assessment, enhancing both accuracy and applicability un- 662 ferent reaction types may significantly reduce thread diver-658 der tight time constraints.

There is further potential for optimization in the program. 665 outcomes, will be automated in future settings.

663 gence. Additionally, the geometric importance for variance 664 reduction, currently adjusted manually based on each run's

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